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Minimal Coupling of the Kalb–Ramond Field to a Scalar Field

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We study the direct interaction of an antisymmetric Kalb–Ramond field with a scalar particle derived from a gauge principle. The method outlined in this paper to define a covariant derivative is applied to a simple model leading to a linear coupling between the fields. Although no conserved Noether charge exists, a conserved topological current comes out from the antisymmetry properties of the Kalb–Ramond field. Some interesting features of this current are pointed out. Possible applications of our results to cosmology and to the theory of three-dimensional Josephson junction arrays are envisaged.

KEY WORDS: Kalb–Ramond field; minimal coupling; topological currents; Noether symmetries.

1. INTRODUCTION

Space-time noncommutativity is one of the key new ideas which follows from recent developments in string and matrix theory (Lizzi and Szabo, 1998). Noncommutativity implies general covariance and, therefore, it seems likely that a noncommutative Yang–Mills theory is a good candidate for a unified and potentially renormalizable theory of the fundamental interactions including gravity.

Whereas the structure of the space-time becomes noncommutative, we can describe it, in analogy to quantum phase space, in terms of the algebra generated by noncommuting coordinates:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}(x), \tag{1}$$

with $\theta^{\mu\nu}$ an antisymmetric tensor (Lizzi *et al.*, 2002).

Antisymmetric tensor fields are widely used in string models (Cremmer and Scherk, 1974; Kalb and Ramond, 1974; Nambu, 1976) as well as in some supersymmetric theories. For example, they appear naturally in $N = 2$ extended

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supersymmetry as auxiliary fields (being the highest dimension fields in a supermultiplet) and in the 11-dimensional formulation of $N = 8$ extended supergravity in which they are dynamical fields. Furthermore, in quantum gravity, antisymmetric tensor fields appear as Lorentz ghost fields (Townsed and van Niewenhuizen, 1977). It is, therefore, quite important to study the dynamics of such fields and, especially, their coupling to matter (scalar or fermion particles).

The best studied example of an antisymmetric tensor field is the electromagnetic field strength $F_{\mu\nu}$, whose dynamics is very well known (see, for example, Jackson, 1963). The coupling of the electromagnetic field to matter fields proceeds, usually, through the *gauge principle* with the aid of the vector potential A_μ (a massless rank 1 field): interaction is introduced in the theory by requiring local gauge invariance for the matter fields. In the electromagnetic case, the gauge transformations for the vector potential are $A_\mu(x) \to A_\mu(x) + \partial_\mu \alpha(x)$, where $\alpha(x)$ is an arbitrary x-dependent function, and the interaction with a charged scalar field φ (with charge e) is obtained by replacing the usual derivative ∂_μ with the covariant derivative D_μ in the free field Lagrangian:

$$\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu. \tag{2}$$

In fact, with this substitution, the total Lagrangian (free fields + interaction) becomes invariant under the local transformation:

$$\varphi(x) \to e^{-ie\alpha(x)}\varphi(x). \tag{3}$$

As a consequence of gauge invariance, by virtue of the Noether theorem, electric charge conservation is obtained. Note, however, that such a minimal coupling prescription is not the only possible one; for example, magnetic moment interaction is described by a Lagrangian term involving directly the physical field strength $F_{\mu\nu}$ rather than the gauge-dependent vector potential.

Several papers have appeared (Botta Cantcheff, 2002) in which the generalization of the gauge principle to abelian rank 2 antisymmetric fields (Kalb–Ramond fields) is studied. However the interaction of a scalar or fermion particle with a Kalb–Ramond field usually proceeds through the coupling with the Maxwell field (Hari Dass and Shajesh, 2002). Indeed, the matter particles interact with the electromagnetic field coupled to a Kalb–Ramond field so that only an indirect interaction is allowed.

While the coupling between such fields and matter fields can always be introduced by adding an ad hoc term in the complete Lagrangian without invoking a gauge principle, the main problem with an interaction generated by (abelian) gauge group transformation comes from the difficulty to construct a covariant derivative from rank 2 gauge fields in analogy with the electromagnetic case. The present work is aimed to further study such a problem by considering, for simplicity, massless fields.

In view of some applications considered below in the paper, in the next section we briefly review the dynamics of an antisymmetric tensor field and point out its substantial equivalence (in the massless case) with that of a (real) scalar field. Instead in section 3 we give a procedure to couple a scalar particle with a Kalb–Ramond field through a gauge principle with a linear coupling; some comments on quadratic coupling are also reported. Finally, in section 4, we study the dynamics of the interacting fields pointing out some peculiar features, and in section 5 we outline some interesting applications, while in the last section we give our conclusions and outlook.

2. DYNAMICS OF A FREE KALB–RAMOND FIELD

Let us consider a Maxwell-like gauge theory where the role of the vector potential A_μ is played by an antisymmetric tensor field $\theta_{\mu\nu}$. Its dynamics is described by the following Lagrangian:

$$\mathcal{L}_\theta = -\frac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu}, \tag{4}$$

where the field strength $H_{\lambda\mu\nu}$ is defined by

$$H_{\lambda\mu\nu} \equiv \partial_\lambda \theta_{\mu\nu} + \partial_\mu \theta_{\nu\lambda} + \partial_\nu \theta_{\lambda\mu}. \tag{5}$$

The equations of motion for the free $\theta_{\mu\nu}$ field are, then, similar to the Maxwell equations and read as follows:

$$\partial_\lambda H^{\lambda\mu\nu} = 0. \tag{6}$$

The Lagrangian in Eq. (4) is invariant under the gauge transformations:

$$\theta_{\mu\nu} \to \theta'_{\mu\nu} = \theta_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu, \tag{7}$$

where Λ_ν is an arbitrary vector field. This gauge freedom can be used to simplify the writing of the field equations. Indeed, for example, by using the "generalized" Lorentz condition $\partial^\mu \theta_{\mu\nu} = 0$ we find that $\theta_{\mu\nu}$ satisfies the ordinary wave equation:

$$\Box \theta_{\mu\nu} = 0. \tag{8}$$

It is easy to prove that the degrees of freedom of the antisymmetric tensor field $\theta_{\mu\nu}$ are just the same as those of a scalar field θ. Let us consider the dual vector field (in the sense of Poincaré) θ_μ defined by

$$\theta_\mu \equiv \frac{1}{6} \epsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \partial^\nu \theta^{\rho\sigma}, \tag{9}$$

which satisfies the Bianchi-type identity:

$$\partial^\mu \theta_\mu = 0. \tag{10}$$

The equations of motion in (6) now become

$$\partial_\mu \theta_\nu - \partial_\nu \theta_\mu = 0, \tag{11}$$

pointing out that θ_μ has to be the gradient of a scalar field θ:

$$\theta_\mu = \partial_\mu \theta. \tag{12}$$

With these changes of variable, the Lagrangian describing the system can be cast in a form similar to that for a scalar field:

$$\mathcal{L}_\theta = \frac{1}{2}\partial_\mu \theta \partial^\mu \theta, \tag{13}$$

this showing that the dynamics of a massless antisymmetric tensor field $\theta_{\mu\nu}$ is completely equivalent (on the classical level[4]) to that of a massless real scalar field θ. Note, however, that Eq. (12) is a direct consequence of the free field equations of motion (11), so that the mentioned equivalence holds true only in the case of noninteracting fields.

3. GAUGE PRINCIPLE WITH A LINEAR COUPLING

Let us consider a charged scalar field ϕ described by the usual Lagrangian:

$$\mathcal{L}_\phi = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - V(\phi), \tag{14}$$

where $V(\phi)$ is a given scalar potential including, eventually, a mass term. In this section we study direct interaction of this field with a Kalb–Ramond field $\theta_{\mu\nu}$ generated by possible gauge group transformations leading to linear coupling with the $\theta_{\mu\nu}$ field.

In analogy with the electromagnetic case, we introduce a covariant derivative D_μ as follows:

$$i D_\mu = i\partial_\mu + k X_\mu, \tag{15}$$

where X_μ is a rank 1 tensor to be defined in terms of the $\theta_{\mu\nu}$ field and k is a suitable coupling constant.

The Lagrangian describing the scalar field ϕ interacting with $\theta_{\mu\nu}$ is, therefore,

$$\mathcal{L}_{\text{int}} = (D^\mu \phi)^\dagger (D_\mu \phi) + V(\phi). \tag{16}$$

Obviously the Lagrangian in Eq. (16) must be invariant under the gauge transformation in Eq. (7) so that the transformation properties of X_μ (and the corresponding ones for the field ϕ) are crucial in the identification of X_μ itself.

Note, however, that the field X_μ cannot be written as a gradient of a given function $\alpha(x)$, to have a "genuine" interaction Lagrangian in Eq. (16). Indeed, let us assume that

$$X_\mu = \partial_\mu \alpha, \tag{17}$$

[4] For the quantum equivalence, see, for example, Duff and van Nieuwenhuizen (1980), Mecklenburg and Mizrachi (1984), and Pathinayake *et al.* (1988).

and consider a local phase transformation for the scalar field ϕ:

$$\phi \rightarrow \Phi = \phi e^{ik\alpha}. \tag{18}$$

Substitution into Eq. (16) immediately leads to

$$\mathcal{L}_{\text{int}} = (D^\mu \phi)^\dagger (D_\mu \phi) + V(\phi) = (\partial^\mu \Phi)^\dagger (\partial_\mu \Phi) + V(\phi), \tag{19}$$

and since Φ and ϕ represent the same physical object, we conclude that the interaction introduced by means of the X_μ field in Eq. (17) is fictitious, because it can be reabsorbed by a phase redefinition of the scalar field.

Starting from the rank 2 tensor $\theta_{\mu\nu}$, the simplest linear choice for X_μ is the following:

$$X_\mu = \partial^\nu \theta_{\mu\nu}. \tag{20}$$

Such a definition for X_μ is, however, useful only when the "generalized" Lorentz condition is not fulfilled, since in this case X_μ is identically zero and no physical interaction appears. Disregarding this gauge choice, the gauge group for the scalar field ϕ is easily obtained. Indeed, by applying the gauge transformation in Eq. (7) to the Lagrangian in (16), we find that it remains unchanged if the scalar field ϕ transforms as follows:

$$\phi \rightarrow \phi' = \phi e^{ik\eta}, \tag{21}$$

with $\eta = \partial^\nu \Lambda_\nu$. Note that the gauge freedom for the $\theta_{\mu\nu}$ field is not altered if in Eq. (7) we choose a divergence-less gauge function Λ_ν. In this case η is zero and Eq. (21) reduces to the unit transformation so that the gauge group underlying the choice in Eq. (20), when it is applicable, acts as the identity on the scalar field ϕ. As a consequence, by means of the Noether theorem, no conserved charge comes out.

The only alternative for X_μ, which is linear in the $\theta_{\mu\nu}$ field, is the following

$$X_\mu = \theta_\mu = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \partial^\nu \theta^{\rho\sigma}. \tag{22}$$

Now we have no limitation on the gauge condition to be used and is a simple task to show that the gauge group associated to the scalar field is, again, the identity. Indeed, it is easy to convince ourselves that X_μ in Eq. (22) is invariant under the gauge transformation in Eq. (7), and thus the Lagrangian in Eq. (16) is automatically invariant when ϕ is unchanged[5]:

$$\phi \rightarrow \phi' = \phi. \tag{23}$$

We remark that the choice (22) for X_μ is not a trivial one since X_μ does not satisfy the constraint (17). In fact, as pointed out in the previous section, the dual field θ_μ can be written as the gradient of θ (see Eq. (16)) only for a massless noninteracting

[5] More in general, we can allow a phase transformation $\phi \rightarrow \phi' = \phi e^{i\beta}$, with a constant β.

Kalb–Ramond field, which is not the present case. It is remarkable that in the limit of no interaction the X_μ field becomes unphysical.

A final remark on the physical dimensions of the coupling constant is in order. We immediately see that, for the cases considered above, k has the dimensions of the inverse of a mass, so that the corresponding theory is nonrenormalizable. We point out that such a property is a peculiar feature of Quantum Gravity, which is a natural framework, however, for the theory developed here.

3.1. Nonlinear Coupling

For completeness, we briefly discuss some particular direct interactions between a scalar field and a Kalb–Ramond field involving nonlinear terms (which are quadratic in the Kalb–Ramond field).

First of all we note that n-linear ($n > 1$) terms in $\theta_{\mu\nu}$ appearing in the covariant derivative have to be gauge invariant in order to assure the invariance of the Lagrangian (neglecting terms which are total divergences). Indeed, when considering the X_μ field where, for example, non gauge invariant bilinear terms in $\theta_{\mu\nu}$ appear, we recognize that, under a gauge transformation (7), two derivatives of two gauge (four vectors) functions Λ_μ come out. In general, the corresponding gauge terms in $(D^\mu\phi)^\dagger(D_\mu\phi)$ cannot be absorbed by a phase transformation (containing only one scalar function) of the scalar field ϕ.

Therefore, simple choices for X_μ involve only the gauge-invariant field $H_{\mu\nu\rho}$ and its dual θ_μ and, as a consequence, the gauge group is the identity.

Some examples of quadratic interactions are as follows:

$$X_\mu = \theta_\nu \partial^\nu \theta_\mu, \tag{24}$$

$$X_\mu = \epsilon^{\alpha\beta\gamma\delta} \partial^\eta H_{\mu\alpha\beta} H_{\gamma\delta\eta}, \tag{25}$$

$$X_\mu = \epsilon_{\mu\nu\alpha\beta} \partial^\nu H^{\alpha\beta\gamma} \theta_\gamma. \tag{26}$$

Note that in the case considered the Eq. (25) (and for similar terms), X_μ vanishes only when $H_{\gamma\delta\eta}$ satisfies its equation of motion in absence of interaction Eq. (6). Finally, we point out that allowing quadratic terms for X_μ as those in Eqs. (24)–(26) (and similar ones), the coupling constant k in the covariant derivative must have the dimensions of the inverse of the 4th power of a mass, so that the nonrenormalizability of the theory is greatly worsened with respect to the linear coupling case.

4. INTERACTING FIELD DYNAMICS

The dynamics of a charged scalar field interacting directly with a Kalb–Ramond field is described by the following Lagrangian:

$$\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) - m^2\phi^\dagger\phi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho}, \tag{27}$$

where the covariant derivative is written as in Eq. (15) and, for simplicity, we have neglected a scalar potential term. By expliciting the X_μ term, we can rewrite Eq. (27) as

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - (m^2 - k^2 X^2)\phi^\dagger \phi$$
$$-ikX_\mu[\phi \partial^\mu \phi^\dagger - \phi^\dagger \partial^\mu \phi] - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho}, \qquad (28)$$

where the second term in the sum accounts for the effective mass acquired by the scalar particle interacting with the Kalb–Ramond field while the following term describes this interaction.

The Euler–Lagrange equation for ϕ immediately follows

$$\partial_\mu \partial^\mu \phi + (m^2 - k^2 X^2)\phi = 2ikX_\mu \partial^\mu \phi + ik\phi \partial^\mu X_\mu. \qquad (29)$$

Note that, in the linear coupling case, the last term vanishes, since $\partial^\mu X = 0$. Instead, the equations of motion for the interacting Kalb–Ramond field are

$$\partial^\sigma H_{\sigma\mu\nu} = J_{\mu\nu}, \qquad (30)$$

with

$$J_{\mu\nu} = 2\partial^\rho \left\{ \left[-ik(\phi \partial_\sigma \phi^\dagger - \phi^\dagger \partial_\sigma \phi) + 2k^2 |\phi|^2 X_\sigma \right] \frac{\partial X^\sigma}{\partial(\partial_\rho \theta^{\mu\nu})} \right\}$$
$$- 2[-ik(\phi \partial_\sigma \phi^\dagger - \phi^\dagger \partial_\sigma \phi) + 2k^2 |\phi|^2 X_\sigma] \frac{\partial X^\sigma}{\partial \theta^{\mu\nu}}. \qquad (31)$$

Given the symmetry properties of the $\theta_{\mu\nu}$ field, it follows that the current $J_{\mu\nu}$ is antisymmetric while exchanging the indices μ and ν:

$$J_{\mu\nu} = -J_{\nu\mu}. \qquad (32)$$

Moreover, from the field Eq. (30) we deduce that, despite of the explicit form of X_μ, $J_{\mu\nu}$ is a conseved current:

$$\partial^\mu J_{\mu\nu} = 0. \qquad (33)$$

Note that such a property follows, again, from the antisymmetry of $H_{\sigma\mu\nu}$.

Let us now consider the explicit interesting case in which X_μ is given by Eq. (22). After some algebra, we obtain the following expression for the current:

$$J_{\mu\nu} = \partial^\sigma T_{\sigma\mu\nu} \qquad (34)$$

with

$$T_{\sigma\mu\nu} = -2(ik\epsilon_{\sigma\rho\mu\nu}\phi^\dagger \partial^\rho \phi + k^2 |\phi|^2 H_{\sigma\mu\nu}). \qquad (35)$$

It is remarkable that, in the case considered, the current $J_{\mu\nu}$ is the gradient of an antisymmetric rank 3 tensor $T_{\sigma\mu\nu}$. Indeed, as a consequence of this, we have that

Eq. (33) holds independently of the equations of motion (by taking the divergence of Eq. (34) we obtain a vanishing R.H.S due to the symmetry properties of $T_{\sigma\mu\nu}$), so that $J_{\mu\nu}$ is a conserved topological current. Moreover on substituting Eq. (34) into Eq. (30), we find that

$$\partial^{\mu}\tilde{H}_{\mu\rho\sigma} = 0 \tag{36}$$

with

$$\tilde{H}_{\mu\rho\sigma} = H_{\mu\rho\sigma} - T_{\mu\rho\sigma}, \tag{37}$$

i.e., the novel field $\tilde{H}_{\mu\rho\sigma}$ can be interpreted as a "dressed" Kalb–Ramond field satisfying the free field equation (36).

4.1. The Symmetry Group of the Topological Current

Let us now turn back to the topological current $J_{\mu\nu}$ in Eqs. (34) and (35) and write it as sum of a term $J^0_{\mu\nu}$, which does not depend explicitly on the Kalb–Ramond field, and a term $J^1_{\mu\nu}$, which vanishes for vanishing $H_{\mu\nu\rho}$:

$$J_{\mu\nu} = -2k\left(J^0_{\mu\nu} + kJ^1_{\mu\nu}\right) \tag{38}$$

$$J^0_{\mu\nu} = \epsilon_{\alpha\beta\mu\nu}(\partial^{\alpha}\varphi^{\dagger})(\partial^{\alpha}\varphi), \tag{39}$$

$$J^1_{\mu\nu} = \partial^{\alpha}(|\varphi|^2 H_{\alpha\mu\nu}). \tag{40}$$

The "free" current $J^0_{\mu\nu}$ remains invariant if we perform a rotation with an imaginary angle $i\alpha$ of the fields φ, φ^{*}[6]:

$$\begin{pmatrix}\varphi\\ \varphi^{*}\end{pmatrix} \rightarrow \begin{pmatrix}\varphi'\\ \varphi^{*\prime}\end{pmatrix} = \begin{pmatrix}\cos i\alpha & \sin i\alpha\\ -\sin i\alpha & \cos i\alpha\end{pmatrix}\begin{pmatrix}\varphi\\ \varphi^{*}\end{pmatrix} \tag{42}$$

However the term $J^1_{\mu\nu}$, and hence the whole current $J_{\mu\nu}$, is not invariant under the group defined above, so that the corresponding topological symmetry can be only viewed as an approximate invariance of the conserved current in the limit of a small coupling constant (the $J^1_{\mu\nu}$-term is quadratic in k). The general transformation for the scalar field φ leaving invariant the whole current $J_{\mu\nu}$ is the following:

$$\varphi \rightarrow \varphi' = \varphi e^{i(\alpha|\varphi|^2+\beta)}, \tag{43}$$

[6] More in general, the current $J^0_{\mu\nu}$ is left unchanged if the fields φ, φ^{*} undergo the following linear transformation:

$$\begin{pmatrix}\varphi\\ \varphi^{*}\end{pmatrix} \rightarrow \begin{pmatrix}\varphi'\\ \varphi^{*\prime}\end{pmatrix} = \begin{pmatrix}a & b\\ b^{*} & a^{*}\end{pmatrix} + \begin{pmatrix}\varphi\\ \varphi^{*}\end{pmatrix} + \begin{pmatrix}c\\ c^{*}\end{pmatrix} \tag{41}$$

with a, b, c three arbitrary complex numbers satisfying the constraint $|a|^2 - |b|^2 = 1$. Note that, since the determinant of the transformation matrix is equal to 1, the group of transformations defined in (41) is that of *equiaffinities* in the complex plane, which is a subgroup of $SL(2, C)$.

with α, β two real parameters, and, in the limit $\alpha = 0$, we recover the simple global phase transformation. However, since Eq. (43) is highly nonlinear for $\alpha \neq 0$, it is not evident the physical meaning corresponding to the transformation in Eq. (43).

5. APPLICATIONS

We here consider two (between others) possible applications of our results, namely cosmology and condensed matter physics (Josephson junction).

In Kalb–Ramond cosmologies (Kao, 1992; Stein-Schabes and Gleiser, 1986) phenomenological limits on some relevant parameters mainly come from axion bounds (Durrer and Sakellariadou, 2000; Giovannini, 1999) by invoking the equivalence between the degrees of freedom of a massless antisymmetric tensor field and those of a massless scalar field (axion). However as pointed out in section 2, this equivalence holds true only in the case of noninteracting fields, so that axion bounds should be extended with care to Kalb–Ramond fields. Such fields, however, can also account for a torsion term in the evolutionary dynamics of the universe (Gorbatove *et al.*, 2002; Kao, 1993; Kar *et al.*, 2002; SenGupta and Sinha, 2001) and its coupling to fermions could result in a helicity flip for them mediated by parity violation (Mukhopadhyaya, 2002a,b). The coupling to cosmologically relevant scalar fields, instead, has not been emphasized in the literature, mainly because of a lacking of a minimal coupling prescription, although the interaction of a Kalb–Ramond with a scalar field may be mediated by the electromagnetic interaction (Hari Dass and Shajesh, 2002).

In particular the direct coupling of the inflaton (or dilaton) to Kalb–Ramond torsion field may result in a short period of anisotropy in the very early stages of the expanding inflationary universe (Di Grezia *et al.*, 2003). The results obtained here will be analyzed in this context in future papers.

Another relevant application of our results is, instead, in condensed matter physics. In fact it has been shown that charge fluctuations around a given ground state in a three-deminsional Josephson junction array can be represented in terms of an antisymmetric Kalb–Ramond gauge field $\theta_{\mu\nu}$ (Diamantini *et al.*, 1995, 1996; Dorey and Movromatos, 1990; Kovner and Rosenstein, 1990; Semenoff and Weiss, 1990). This field is coupled to a Maxwell gauge field A_μ by means of a topological mass term in the BF Model (Allen *et al.*, 1991; Balachandran *et al.*, 1982, 1994):

$$\mathcal{L}_{\mathrm{BF}} = -\frac{1}{12g^2} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{\kappa}{4\pi} \theta_{\mu\nu} \epsilon^{\mu\nu\lambda\rho} F_{\lambda\rho} - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu}. \qquad (44)$$

Here A_μ describes an ordinary photon with field strength given by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; as noted in Diamantini *et al.* (1996), the coupling in Eq. (44) induces a topological mass for the photon:

$$m = \frac{eg\kappa}{\pi}, \qquad (45)$$

which represents the plasma frequency of Josephson junction array. However in that paper the interaction of the Kalb–Ramond field with Cooper pairs in the junction array has not been considered, and we observe that the minimal coupling with scalar degrees of freedom analyzed here has a natural framework in the BCS theory (Tinkham, 1975), where only the singlet state of Cooper pairs are taken into account. At a first look, such a coupling would result in an effective topological mass for the Cooper pairs, changing the gap in their energy spectrum. However, since the pairs have a large mass, which is not observable if the junction is in its superconducting phase, this spectrum change would not influence the physical behavior of the three-dimensional Josephson junction array. Instead the induced effective mass for the Kalb–Ramond field, which is massless at the beginning (i.e., without interaction), has far-reaching implications. In fact by studying the Josephson junction array with a simple model of two interacting fluids (the charge fluctuation fluid and the Cooper pair one), we find that charge fluctuations acquire mass and this could lead to a significant change of the superconductivity properties. In particular, by computing the free energy in this model, although no relevant modification of the transport properties would be in order, we expect an improvement of the superconductivity properties and a sensible change of the critical temperature. This subject will be addressed in detail in a future work.

6. CONCLUSIONS

In this paper we have introduced a direct interaction between a scalar particle and a Kalb–Ramond field by defining a covariant derivative D_μ, where an appropriate auxiliary vector field X_μ (depending on the Kalb–Ramond field) appears. Several possible choices for X_μ have been studied, leading to linear or quadratic coupling with the scalar field. In the simple, viable models considered here, the gauge group underlying the theory is the identity, so that no conserved Noether charge exists. However, because of the antisymmetry properties of the Kalb–Ramond field, a conserved (antisymmetric) topological current arises in the simplest model, which appears in the equations of motion for $H_{\mu\nu\rho}$. Since this current is a divergence of a rank 3 antisymmetric tensor, it is possible to define a “dressed” Kalb–Ramond field strength, obeying the free field equations, which describe the dynamics of the interacting field.

Some possible applications of our results are concerned with the theory of gravity, where a zero-mass Kalb–Ramond field is the source of torsion in Einstein–Cartan theory (SenGupta and Sur, 2001). Moreover, in recent years, it has been pointed out that the presence of Kalb–Ramond fields in the background space-time leads to several interesting astrophysical and cosmological phenomena like cosmic optical activity and neutrino helicity flip (Gorbatov *et al.*, 2002; Kar *et al.*, 2002; SenGupta and Sinha, 2001). This motivates the study of some important problems related to the standard Friedman–Robertson–Walker cosmological model in light of Kalb–Ramond cosmology (Lizzi *et al.*, 2002; SenGupta and Sur, 2002) in an inflationary framework, where the coupling to a scalar field is crucial.

Further important implications of the results obtained here are expected in the theory of three-dimensional Josephson junction arrays, where an effective mass for charge fluctuations arises from the interaction with Cooper pairs in the BCS theory. This would lead to an improvement of the superconductivity properties of the junction itself, along with a change of its critical temperature.

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